Supplementary Document on Real-Time High-Quality Specular Highlight Removal using Efficient Pixel Clustering

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I. Minimum-Maximum Chromaticity Space

In this section, we mathematically analyze the minimum-maximum chromaticity space used in the paper.

First, let us recall that the pseudo diffuse chromaticity $\Lambda^{\text{psf}}(x)$ is typically estimated from the pseudo specular-free image $\mathsf{I}^{\text{psf}}(x)$ as

$$\Lambda^{\text{psf}}(x) = \frac{\mathsf{I}^{\text{psf}}(x)}{\mathsf{I}_{\text{r}}^{\text{psf}}(x) + \mathsf{I}_{\text{g}}^{\text{psf}}(x) + \mathsf{I}_{\text{b}}^{\text{psf}}(x)}. \tag{1}$$

Now, for convenience, let us assume that the diffuse chromaticity $\Lambda^{\text{psf}}(x)$ can be rewritten as

$$\Lambda^{\text{psf}}(x) = [\Lambda_{\text{max}}^{\text{psf}}(x), \Lambda_{\text{int}}^{\text{psf}}(x), \Lambda_{\text{min}}^{\text{psf}}(x)]^{\text{T}}, \tag{2}$$

where $\Lambda_{\text{int}}^{\text{psf}}(x)$ is an intermediate chromaticity value that lies between $\Lambda_{\text{max}}^{\text{psf}}(x)$ and $\Lambda_{\text{min}}^{\text{psf}}(x)$

$$\Lambda_{\text{max}}^{\text{psf}}(x) \geq \Lambda_{\text{int}}^{\text{psf}}(x) \geq \Lambda_{\text{min}}^{\text{psf}}(x). \tag{3}$$

From (1), we already know that $\Lambda^{\text{psf}}(x)$ is normalized to the unit interval $[0, 1]$, since it is computed by the normalization of $\mathsf{I}^{\text{psf}}(x)$. Therefore, we can also state that

$$\Lambda_{\text{max}}^{\text{psf}}(x) + \Lambda_{\text{int}}^{\text{psf}}(x) + \Lambda_{\text{min}}^{\text{psf}}(x) = 1. \tag{4}$$

From the equation above, the maximum chromaticity value $\Lambda_{\text{max}}^{\text{psf}}(x)$ can be rewritten as

$$\Lambda_{\text{max}}^{\text{psf}}(x) = 1 - (\Lambda_{\text{int}}^{\text{psf}}(x) + \Lambda_{\text{min}}^{\text{psf}}(x)). \tag{5}$$

Experimentally, we have projected pixels of different images in the minimum-maximum chromaticity space and we could see that their projections lie inside a triangle similar to the one shown in Figure 1. To determine the position of the vertices of that triangle, let us estimate the extreme (highest and lowest) values of both minimum and maximum chromaticity intensities that can be computed from (1).

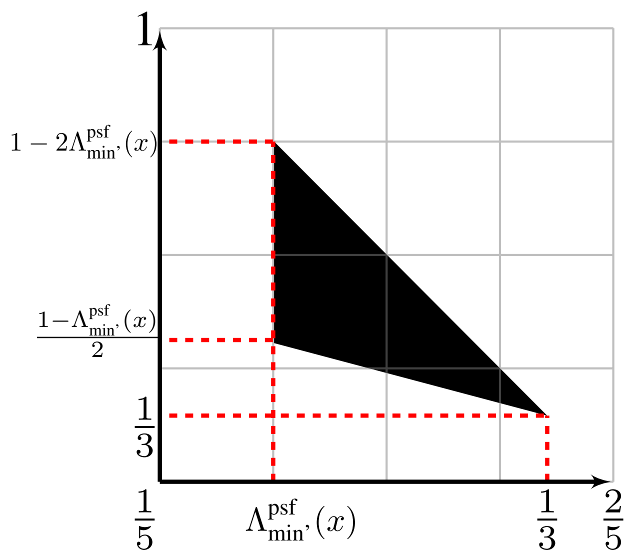

Fig. 1. The minimum-maximum chromaticity space has the form of a triangle shape that can be defined in terms of the lowest minimum chromaticity value $\Lambda_{\text{min'}}^{\text{psf}}(x)$. In this illustration, the x-axis contains minimum chromaticity values and the y-axis contains maximum chromaticity values.

The highest minimum chromaticity value that can be estimated from (1) is generated when $\Lambda_{\text{max}}^{\text{psf}}(x) = \Lambda_{\text{int}}^{\text{psf}}(x) = \Lambda_{\text{min}}^{\text{psf}}(x)$. By replacing two of those terms in (5), we have:

$$\begin{aligned} \Lambda_{\text{min}}^{\text{psf}}(x) &= 1 - (\Lambda_{\text{min}}^{\text{psf}}(x) + \Lambda_{\text{min}}^{\text{psf}}(x)) \\ 3\Lambda_{\text{min}}^{\text{psf}}(x) &= 1 \\ \Lambda_{\text{min}}^{\text{psf}}(x) &= 1/3. \end{aligned} \tag{6}$$

So, the highest $\Lambda_{\text{min}}^{\text{psf}}(x)$ that can be computed from $\mathsf{I}^{\text{psf}}(x)$ is equivalent to the lowest $\Lambda_{\text{max}}^{\text{psf}}(x)$, that is $\frac{1}{3}$. As shown in Figure 1, the pair $(\frac{1}{3}, \frac{1}{3})$ defines the position of one of the vertices of the triangle that encloses the minimum-maximum chromaticity space.

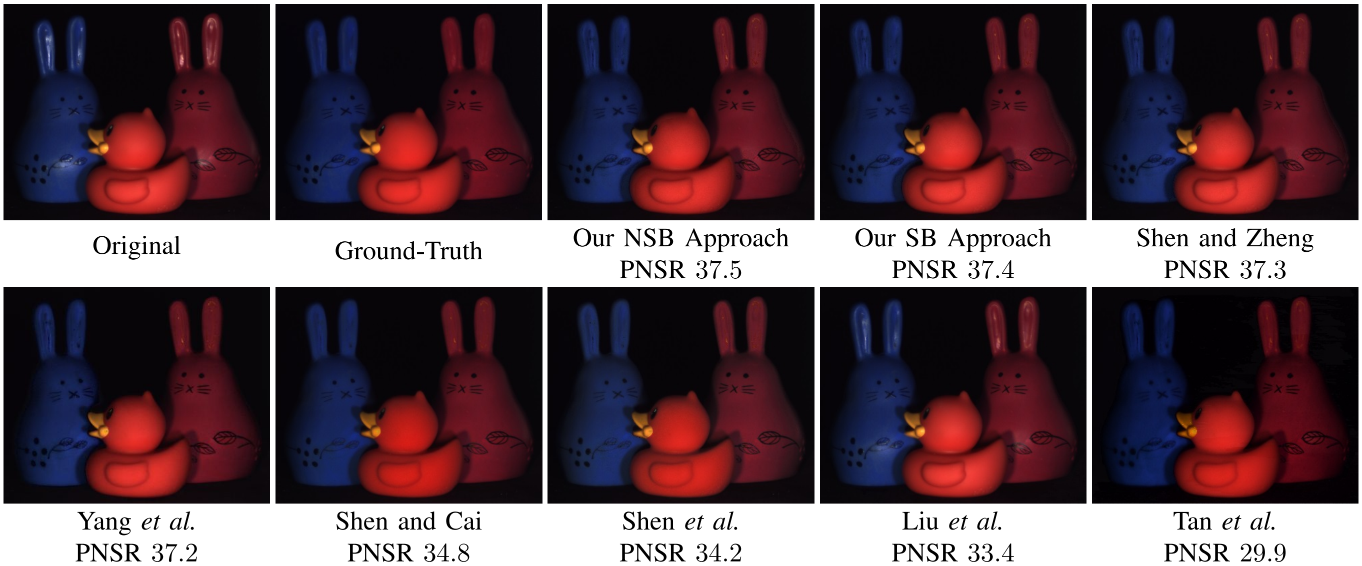

Fig. 2. A visual comparison between different specular highlight removal techniques for the *Animals* image.

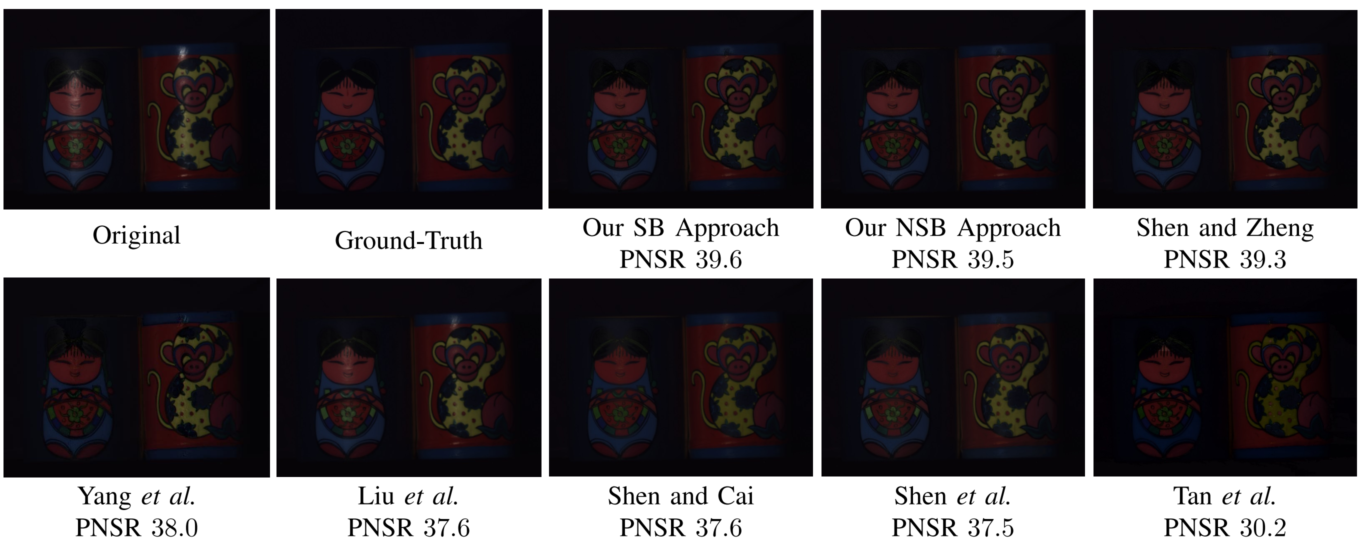

Fig. 3. A visual comparison between different specular highlight removal techniques for the *Cups* image.

Now, let us assume $\Lambda^{\mathsf{psf}}_{\mathsf{min'}}(x)$ to be the lowest minimum chromaticity value that can be estimated from $\mathsf{I}^{\mathsf{psf}}$. To prevent the minimum chromaticity channel to be zero, let us recall that we add the mean value $\overline{\mathsf{I}^{\mathsf{min}}}$ to each pixel of $\mathsf{I}^{\mathsf{psf}}$. In this sense, the lowest minimum chromaticity value that can be computed from a relevant pixel in $\mathsf{I}^{\mathsf{psf}}$ is

$$\Lambda^{\mathsf{psf}}_{\mathsf{min'}}(x) = \frac{\overline{\mathsf{I}^{\mathsf{min}}}}{\mathsf{I}^{\mathsf{psf}}_{\mathsf{r}}(x) + \mathsf{I}^{\mathsf{psf}}_{\mathsf{g}}(x) + \overline{\mathsf{I}^{\mathsf{min}}}}, \tag{7}$$

assuming that the lowest color channel of $\mathsf{I}^{\mathsf{psf}}$ is in the blue color channel, only for the purpose of the explanation.

On the basis of $\Lambda^{\mathsf{psf}}_{\mathsf{min'}}(x)$, we can compute the highest and the lowest maximum chromaticity values that can be estimated for a pixel when $\Lambda^{\mathsf{psf}}_{\mathsf{min}}(x) = \Lambda^{\mathsf{psf}}_{\mathsf{min'}}(x)$. This will help us to further estimate the positions of the remaining vertices of the triangle shown in Figure 1.

The highest maximum chromaticity value that can be computed from (1) is estimated when $\Lambda^{\mathsf{psf}}_{\mathsf{int}}(x) = \Lambda^{\mathsf{psf}}_{\mathsf{min'}}(x)$. Then

$$\begin{aligned} \Lambda^{\mathsf{psf}}_{\mathsf{max}}(x) &= 1 - (\Lambda^{\mathsf{psf}}_{\mathsf{min'}}(x) + \Lambda^{\mathsf{psf}}_{\mathsf{min'}}(x)) \\ \Lambda^{\mathsf{psf}}_{\mathsf{max}}(x) &= 1 - 2\Lambda^{\mathsf{psf}}_{\mathsf{min'}}(x). \end{aligned} \tag{8}$$

Likewise, the lowest maximum chromaticity value that can be estimated for a pixel with $\Lambda^{\mathsf{psf}}_{\mathsf{min}}(x) = \Lambda^{\mathsf{psf}}_{\mathsf{min'}}(x)$ requires $\Lambda^{\mathsf{psf}}_{\mathsf{int}}(x) = \Lambda^{\mathsf{psf}}_{\mathsf{max}}(x)$ and $\Lambda^{\mathsf{psf}}_{\mathsf{int}}(x) \neq \Lambda^{\mathsf{psf}}_{\mathsf{min'}}(x)$. Hence

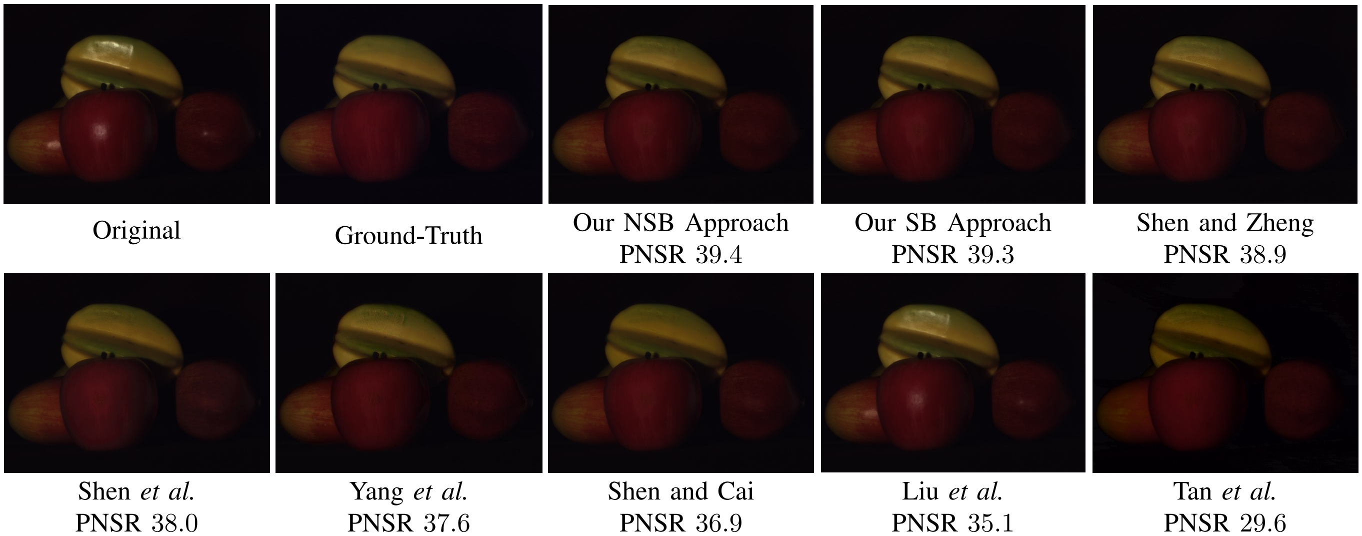

Fig. 4. A visual comparison between different specular highlight removal techniques for the *Fruit* image.

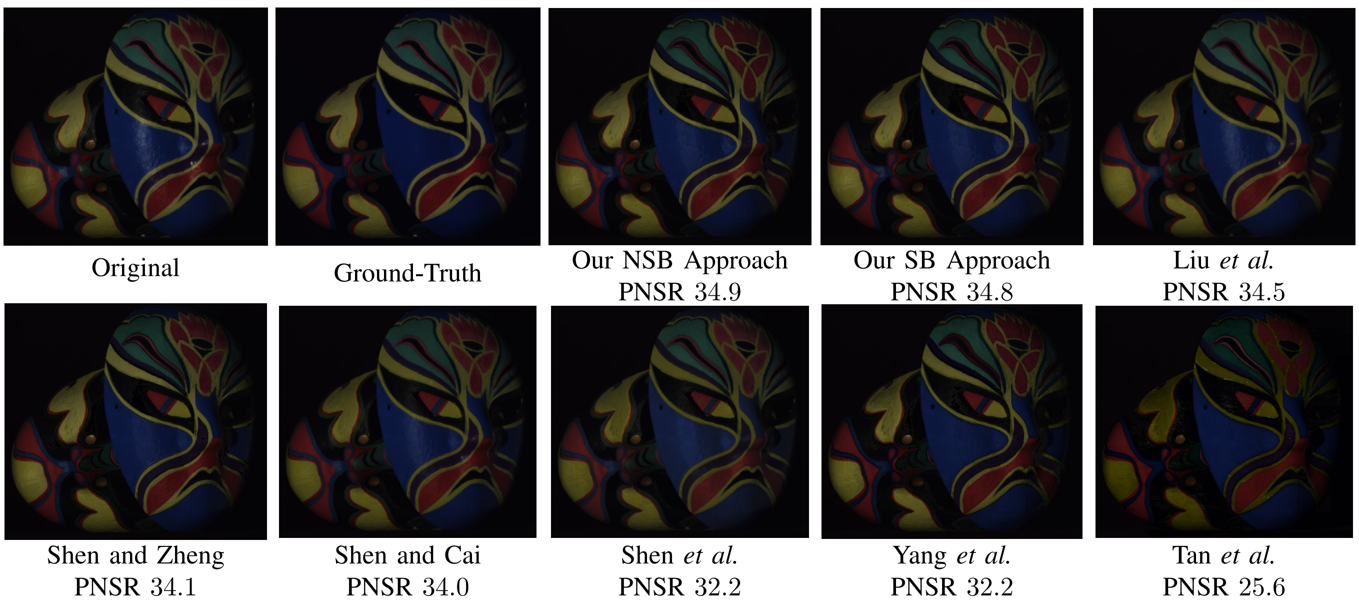

Fig. 5. A visual comparison between different specular highlight removal techniques for the *Masks* image.

$$\begin{aligned}
\Lambda^{\text{psf}}_{\text{max}}(x) &= 1 - (\Lambda^{\text{psf}}_{\text{max}}(x) + \Lambda^{\text{psf}}_{\text{min'}}(x)) \\
2\Lambda^{\text{psf}}_{\text{max}}(x) &= 1 - \Lambda^{\text{psf}}_{\text{min'}}(x) \\
\Lambda^{\text{psf}}_{\text{max}}(x) &= \frac{1 - \Lambda^{\text{psf}}_{\text{min'}}(x)}{2}.
\end{aligned} \tag{9}$$

In this sense, the pairs of points $(1 - 2\Lambda^{\text{psf}}_{\text{min'}}(x), \Lambda^{\text{psf}}_{\text{min'}}(x))$ and $(\frac{1-\Lambda^{\text{psf}}_{\text{min'}}(x)}{2}, \Lambda^{\text{psf}}_{\text{min'}}(x))$ define the positions of the remaining vertices in Figure 1. Therefore, we have shown that any pixel projected in the minimum-maximum chromaticity space has, necessarily, a pair of minimum and maximum chromaticity values that lies inside the triangle shown in Figure 1.

II. Additional Results

In Figures 2, 3, 4 and 5, we provide an additional visual comparison between our approach and related work for the four images available in the standard specular highlight removal dataset.

In Figure 6, we show how the techniques perform the specular highlight removal for a real image captured in an uncontrolled environment. Unfortunately, all of the evaluated techniques (including ours) are still unable to properly minimize the specular highlights of the image without making the specular region too dark. As can be seen in Figure 6, the specular highlight regions of the fruits are put in black by all

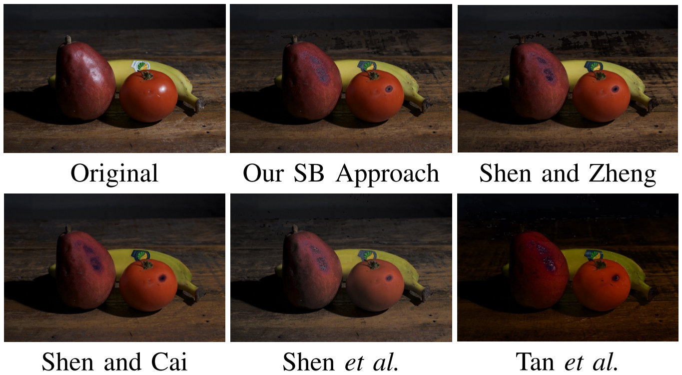

Fig. 6. A visual comparison between different specular highlight removal techniques for a real image with fruits. Images are courtesy of Flickr user @thewazir.

Fig. 7. A visual comparison between different specular highlight removal techniques for an image with a large specular highlight region.

the techniques. Moreover, none of the techniques are able to distinguish that the white sticker on top of the banana is not a specular highlight region, and so must be kept white even after the specular highlight removal.

In Figure 7, we show how the techniques perform the specular highlight removal for an image with a large specular highlight region. Again, none of the evaluated techniques can handle this challenging task efficiently.